The -ve sign shows that force is attractive in nature b/w the pt. charge & image charge.

The resultant force can be expressed as:-

$$F = \frac{-1}{4\pi\epsilon_0} \frac{a\,dq^2}{[d^2 - a^2]^2} \quad \text{—— (15)}$$

Now two cases may arises:-

1) Case:- I :- When the point charge is at large distance i.e; $d \gg a$. In such condⁿ, the attractive force acting b/w the pt charge & image charge is changed which can be expressed as:-

$$F = \frac{-1}{4\pi\epsilon_0} \frac{adq^2}{d^4}$$

$$F = -\frac{1}{4\pi\epsilon_0} \frac{aq^2}{d^3}$$

$$\boxed{F \propto -\frac{1}{d^3}}$$

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From above it is clear that if the pt charge is at large distance from the conducting sphere then the force is inverse cube force & attractive in nature.

2) Case 2 :- When the pt. charge is near to the conducting sphere. Let us consider a distance 'x' which is very less than the radius of the conducting sphere.

Although $d = a + x$

then $$F = \frac{-1}{4\pi\epsilon_0} \frac{aq^2(a+x)}{[(a+x)^2 - a^2]^2}$$

$$F = \frac{-1}{4\pi\epsilon_0} \frac{aq^2(a+x)}{[a^2+x^2+2ax-a^2]^2}$$

$$F = \frac{-1}{4\pi\epsilon_0} \frac{aq^2(a+x)}{(x^2+2ax)^2}$$

since, $a >>> x$ So above eq^n reduced in the form of :-

$$F = \frac{-1}{4\pi\epsilon_0} \frac{q^2a^2}{4a^2x^2}$$

$$\boxed{F = \frac{-1}{4\pi\epsilon_0} \frac{q^2}{4x^2}}$$ or $F \propto \frac{1}{x^2}$

From above it is clear that the force is inverse square force the pt. charge is nearer to the conducting sphere then it always follow inverse square law.

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(C) Electric Field Intensity and potential due to insulated sphere:-

In the case of insulated surface following condition must be satisfied :-

1. The distribution of potential at the surface of sphere remain uniform.

2. The induced charge of the insulated sphere remains zero.

For this, let us consider a point charge (+q) which is situated at point A and at a distance of 'd' from the centre of insulating sphere. Let the image charge $\left(-\frac{qa}{d}\right)$ which is placed at a point I at a distance of a^2/d from the centre of sphere. Let an another Image charge $(+qa/d)$ is placed at the centre of the insulating sphere.

The distribution of potential

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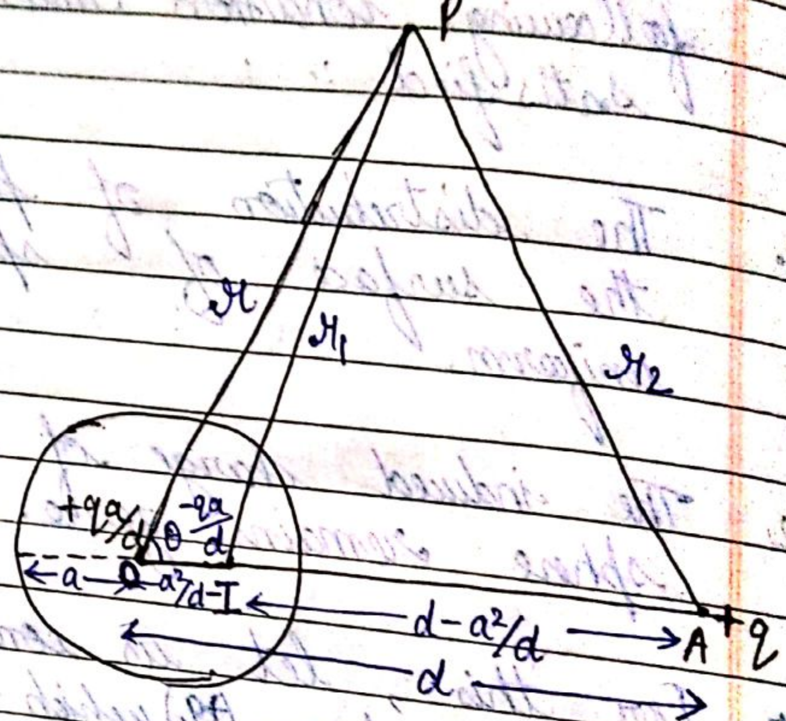
at the surface of Insulated sphere remains zero. Then the potential at point P can be expressed as :-



$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{qa}{rd} - \frac{qa}{r_1 d} + \frac{q}{r_2}\right]$$

ΔOPI where $r_1 = \left[r^2 + \frac{a^4}{d^2} - \frac{2ra^2}{d}\cos\theta\right]^{1/2}$

& $r_2 = \left[r^2 + d^2 - 2rd\cos\theta\right]^{1/2}$

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{qa}{rd} - \frac{qa}{d\left(r^2 + \frac{a^4}{d^2} - \frac{2ra^2}{d}\cos\theta\right)^{1/2}} + \frac{q}{(r^2 + d^2 - 2rd\cos\theta)^{1/2}}\right]$$

Now, the electric field intensity due to the radial component can be expressed as :-

$$E_r = \frac{-\delta V}{\delta r} = \frac{1}{4\pi\epsilon_0}\left[\frac{qa}{r^2 d} - \frac{qa\left(r - \frac{a^2}{d}\cos\theta\right)}{d\left(r^2 + \frac{a^4}{d^2} - 2r\frac{a^2}{d}\cos\theta\right)^{3/2}} + \frac{q(r - d\cos\theta)}{(r^2 + d^2 - 2rd\cos\theta)^{3/2}}\right] \quad (2)$$

This is the required electric field intensity due to radial component.

Now, for Orientation component, the electric field intensity can be expressed as :-

$$E_\theta = -\frac{1}{r}\frac{\delta V}{\delta \theta}$$

$$E_\theta = \frac{1}{r}\frac{1}{4\pi\epsilon_0}\left[\frac{qa(ra^2/d \sin\theta)}{d(r^2 + a^4/d^2 - 2ra^2/d\cos\theta)^{3/2}} + \frac{qrd\sin\theta}{(r^2 + d^2 - 2rd\cos\theta)^{3/2}}\right]$$

$$E_\theta = \frac{1}{4\pi\epsilon_0}\left[\frac{q a^3 \sin\theta}{d^2\left(r^2 + a^4/d^2 - \frac{2ra^2\cos\theta}{d}\right)^{3/2}}\right.$$

$$\left. + \frac{q d \sin\theta}{\left(r^2 + d^2 - 2rd\cos\theta\right)^{3/2}}\right] \quad (3)$$

This is the required expression for electric field intensity due to the orientation component.